NOTATION

s, total surface of the thermoelectric element; s_0, s_1, surfaces of the cold and hot contacts, respectively; Σ, thermoelectric element surface outside the contacts; v, volume of the thermoelectric element; u, electric potential; u_0, u_1, potentials of the cold and hot contacts; $\mathbf{i}$ current density vector; σ, λ, electrical conductivity and thermal conductivity; α, absolute thermal emf; $z = \alpha^2\sigma/\lambda$; T, absolute temperature; T_0, T_1, temperatures of the cold and hot contacts; T', temperature at an arbitrary point on the thermoelectric element with no current and at $T_0 \neq T_1$; T", temperature at an arbitrary point on the thermoelectric element with current $T_0 = T_1$; $\vartheta = T - T_0$; $\vartheta' = T' - T_0$; $\vartheta'' = T'' - T_0$; ∇^2, Laplace operator; ∇, Hamiltonian operator; $q_F = -\lambda\nabla T$, conduction heat-transfer vector; $q_{F_1} = -\lambda\nabla T'$; $q_{F_2} = -\lambda\nabla T''$; Q_{F_1}, heat conduction through contact surfaces with no current; Q_J, total power of internal Joule heat sources; φ, fraction of the total power of the internal sources transferred by heat conduction to the surface s_0 at $T_0 = T_1$; $\mu = \mu^* + eu + e\alpha T$; μ^*, chemical potential; e, carrier charge $I = \iint_{s\mu} \mathbf{i} \cdot d\mathbf{s}$, electric current; s_μ, equipotential surface; $Q_{\pi}|_{s_0} = \alpha T_0 I$, Peltier heat absorbed on a cold contact; $Q_{\pi}|_{s_0} = \alpha T_1 I$, Peltier heat generated at a hot contact; Φ, form factor; Q_0, heat removed from cold source; Q_1, heat supplied to hot source.

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GENERALIZED STATIC VOLT–AMPERE CHARACTERISTICS OF THERMORESISTORS

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UDC 536.531

Similarity criteria are obtained for static volt–ampere characteristics of thermoresistors and for thermoresistors included in a circuit. A technique is described for a simplified graphic-analytical design of a circuit with a thermoresistor and rules are given for modeling thermoresistors where the dissipation coefficient varies.

1. Similarity Criteria for Static Volt–Ampere Thermoresistor Characteristics

We begin with the assumption that the temperature T is constant over the entire volume of the thermoresistor, which is approximately true [1, 2] when

$$\mathrm{Bi} \ll 1 \tag{1}$$

(Bi is the Biot number).

We can write the heat-balance equation for a thermoresistor, relating the current i and the voltage u on it with the environment temperature T_e and the dissipation coefficient H:

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$$ui = \frac{u^2}{R_T} = i^2 R_T = H(T - T_e). \tag{2}$$

In general, we can write the following expressions for the quantities R_T and H:

$$R_T = R_0 F_R\left(\frac{T}{B}\right) = R_0 F_R(\Theta), \tag{3}$$

$$H = H_0 F_H\left(\frac{T}{B}, \frac{T_e}{B}\right) = H_0 F_H(\Theta, \Theta_e), \tag{4}$$

where R_0 and H_0 are constants, having the dimensions of resistance and dissipation coefficients, respectively; B is a constant, having the dimensions of temperature; $\Theta = T/B$ and $\Theta_e = T_e/B$ are normalized values of the thermoresistor and environment temperature; and F_R and F_H are functions of the dimensionless argument Θ, giving R_T and H as a function of thermoresistor temperature (H depends, in addition, on the temperature T_e of the surrounding environment and, therefore, on the parameter Θ_e).

The methods of similarity and dimensional analysis are applicable only when the conditions

$$F_R(\Theta) = \text{idem}, \tag{5}$$

$$F_H(\Theta, \Theta_e) = \text{idem} \tag{6}$$

hold for the volt–ampere characteristics being considered.

Condition (6) can be considered to hold in most cases (although approximately), since, first, the dissipation coefficient of thermoresistors depends only slightly on the temperatures T and T_e, and when these vary, e. g., in a range on the order of 100°C, H varies only by some tens of percent [1-3]; secondly, the dependence $H = f(T, T_e)$ is qualitatively the same for different types of thermoresistors (the quantity H increases both with increase of excess temperature $\Delta T = T - T_e$ of the thermoresistor relative to the environment at a given value of T_e and with increase of the environment temperature T_e for a given excess temperature ΔT).

In contrast with H, the thermoresistor resistance R_T can vary by a factor of several tens and even a hundred with change of temperature T over a range on the order of 100°C (e. g., for thermoresistors with a critical temperature [4, 5]), and the law can be different (compare, e. g., the posistor and the ordinary thermoresistor). Therefore, it is very important that condition (5) hold for similarity of the volt–ampere characteristics being compared, and this is true for thermoresistors of a single type and sometimes even for entire classes of thermoresistor [e. g., for thermoresistors $F_R(\Theta) = \exp(1/\Theta) = \text{idem}$].

Assuming that conditions (5) and (6) hold, it is easy to obtain the following expression for the current i and the voltage u on the thermoresistor from Eq. (2):

$$\frac{i^2}{\frac{BH_0}{R_0}} = \frac{F_H(\Theta, \Theta_e)(\Theta - \Theta_e)}{F_R(\Theta)} = F_i(\Theta, \Theta_e), \tag{7}$$

$$\frac{u^2}{BH_0R_0} = F_R(\Theta) F_H(\Theta, \Theta_e)(\Theta - \Theta_e) = F_u(\Theta, \Theta_e). \tag{8}$$

We now introduce the following normalized dimensionless quantities: the voltage $\bar{u}$ on the thermoresistor, the current $\bar{i}$ flowing through it, and the resistance $\bar{R}_T$:

$$\bar{u} = \frac{u}{m_u} = \frac{u}{\sqrt{BH_0R_0}}, \quad \bar{i} = \frac{i}{m_i} = \frac{i}{\sqrt{\frac{BH_0}{R_0}}}, \quad \bar{R}_T = \frac{R_T}{m_R} = \frac{R_T}{R_0}. \tag{9}$$

Here

$$m_u = \sqrt{BH_0R_0}, \quad m_i = \sqrt{\frac{BH_0}{R_0}}, \quad m_R = R_0 \tag{10}$$

are assumed to be the measurement scales of voltage, current, and resistance, respectively.

It follows from Eqs. (7) and (8) that the normalized dimensionless volt–ampere characteristics of "homogeneous" thermoresistors [in the sense of conditions (5) and (6)] are a family of curves dependent only on a single parameter — the normalized environment temperature Θ_e — and for them the similarity parameter is

$$\bar{u} = F(\bar{i}, \Theta_e). \quad (11)$$

This result was obtained in [6-8] for a particular case [H = const, $R_T = A\exp(B/T)$].

Here we note particularly that it was assumed in the derivation [see Eq. (3)] that the thermoresistor resistance as a whole is determined by the temperature T and does not depend on other factors (this provision is not satisfied, e. g., by posistors, whose resistance also depends on the applied voltage, and this gives rise to the so-called "varistor effect" [4]).

2. Similarity Parameters for Thermoresistors Included in an Electrical Circuit

We consider an arbitrary circuit consisting of a voltage source, linear resistances, and a single thermoresistor. The current i flowing through the thermoresistor is

$$i = \frac{u_{eq}}{R_T + r_{eq}}, \quad (12)$$

where the voltage u_{eq} and the resistance r_{eq} can be determined from the equivalent generator theorem [9].

Substituting Eq. (12) into Eq. (2), and performing some simple transformations using Eqs. (3) and (4), we obtain

$$\frac{F_H(\Theta, \Theta_e)(\Theta - \Theta_e)}{F_R(\Theta)}[F_R(\Theta) + \bar{r}_{eq}]^2 = \bar{u}_{eq}^2, \quad (13)$$

where $\bar{u}_{eq}$ and $\bar{r}_{eq}$ are the normalized dimensionless voltage u_{eq} ($\bar{u}_{eq} = u_{eq}/m_u$) and resistance r_{eq} ($\bar{r}_{eq} = r_{eq}/m_R$). It follows from Eq. (13) that the normalized thermoresistor temperatures and, therefore, their resistances $F_R(\Theta)$ are given by the three dimensionless parameters

$$\bar{u}_{eq} = \frac{u_{eq}}{\sqrt{BH_0R_0}}, \quad \bar{r}_{eq} = \frac{r_{eq}}{R_0}, \quad \Theta_e = \frac{T_e}{B} \quad (14)$$

instead of the six original dimensional parameters: R_0, B, H_0, Te, u_{eq}, and r_{eq}. These three parameters also determine the normalized currents $\bar{i}$ through the thermoresistor and the voltages $\bar{u}$ across it, as follows from Eq. (12), which can be converted to the form

$$\bar{i} = \frac{\bar{u}_{eq}}{F_R(\Theta) + \bar{r}_{eq}}.$$

It follows from the above, in particular, that a change in the heat-transfer conditions (in the dissipation coefficient H_0) of the thermoresistor can be modeled by a change in the supply voltage u_0. Here we assume that the temperature dependence of the dissipation coefficient $F_H(\Theta, \Theta_e)$ remains the same and that condition (1) holds.

In fact, in a change of dissipation coefficient from H_{01} to a new value H_{02}, only the parameters $\bar{u}_{eq}$: $\bar{u}_{eq_2} = \bar{u}_{eq_1}\sqrt{H_{01}/H_{02}}$ change. However, the new value of the parameter $\bar{u}_{eq_2}$ can be obtained even for an unchanged quantity $H_0 = H_{01}$ = const by a change in the circuit supply voltage $u_0 : u_{02} = u_{01}\sqrt{H_{01}/H_{02}}$ (since $\bar{u}_{eq}$ is directly proportional to u_0). Thus, an increase in the dissipation coefficient H_0 by a factor of m is equivalent to a decrease by a factor of $\sqrt{m}$ in the circuit supply voltage; the normalized current $\bar{i}$ and voltage $\bar{u}$ on the thermoresistor are the same in the two cases.

We now convert from the normalized quantities $\bar{u}$ and $\bar{i}$ to the ordinary dimensional values u and i. In the case of variation in H_0 the voltage u and the current i (for given values of $\bar{u}$ and $\bar{i}$) depend on H_0 as follows [see Eq. (9)]:

$$u = \bar{u}\sqrt{BH_0R_0}, \quad i = \bar{i}\sqrt{\frac{BH_0}{R_0}}.$$

In the case of variations in u_0, the absolute values of u and i do not depend on u_0 when the normalized values $\bar{u}$ and $\bar{i}$ are given. Taking into account this difference, we can finally formulate a rule for modeling variations in the conditions of heat transfer from thermoresistors: "An increase (or decrease) in the dissipation coefficient of thermoresistors by a factor of m can be modeled by a decrease (or an increase) by a factor of $\sqrt{m}$ in the supply voltage; here the currents and voltages measured in the modeling must be increased (or decreased) by a factor of $\sqrt{m}$, correspondingly."

3. Simplified Graphic-Analytical Circuit Design Procedure with a Thermoresistor under Static Conditions

We now consider the most varied class of thermoresistors — thermistors whose resistance R_T varies with temperature according to the law

$$R_T = R_e \exp\left(\frac{B}{T} - \frac{B}{T_e}\right). \tag{15}$$

When the environment temperature T_e varies over several tens of degrees, and when the thermoresistor excess temperature relative to the environment is on the same order, the dissipation coefficient H of the thermoresistor may be constant, to a first approximation [3]:

$$H = H_0 \approx \text{const.} \tag{16}$$

We now consider the new variable

$$x = \frac{B}{T_e} - \frac{B}{T} \tag{17}$$

in place of the thermistor temperature T. The excess temperature relative to the environment is then equal to

$$T - T_e = \frac{BT_e}{B - xT_e} - T_e = \frac{T_e^2}{B} \cdot \frac{x}{1 - \Theta_e x}. \tag{18}$$

Using Eqs. (15)-(18), we can transform thermoresistor heat-balance equation (2) into the following form:

$$\frac{u^2}{\dfrac{H_0 R_e T_e^2}{B}} = \frac{x}{1 - \Theta_e x} \exp(-x) \tag{19}$$

or

$$\frac{i^2}{\dfrac{H_0 T_e^2}{BR_e}} = \frac{x}{1 - \Theta_e x} \exp x. \tag{20}$$

We consider the normalized dimensionless voltage $\bar{u}$ over the thermoresistor and the normalized current $\bar{i}$ flowing through it:

$$\bar{u} = \frac{u}{T_e \sqrt{\dfrac{H_0 R_e}{B}}}, \quad \bar{i} = \frac{i}{T_e \sqrt{\dfrac{H_0}{BR_e}}}. \tag{21}$$

Here

$$M_u = T_e \sqrt{\frac{H_0 R_e}{B}}, \quad M_i = T_e \sqrt{\frac{H_0}{BR_e}} \tag{22}$$

have been adopted as the voltage and current measurement scales, respectively.

The normalized dimensionless volt—ampere thermistor characteristics are thus given parametrically by the following equations:

$$\bar{u} = \sqrt{\frac{x}{1 - \Theta_e x}} \exp\left(-\frac{x}{2}\right), \quad \bar{i} = \sqrt{\frac{x}{1 - \Theta_e x}} \exp\frac{x}{2}. \tag{23}$$

We shall show that with the above normalization, the dimensionless volt—ampere thermoresistor characteristics are practically independent of the environment temperature over a range of several tens of degrees [only the scales M_u and M_i, into which the quantities T_e and $R_e = f(T_e)$ enter, depend on the environment temperature T_e].

In fact, it follows from Eq. (23) that for a variation in the environment temperature T_e over a range $\pm\Delta T_e$ about its mean value τ_e, the relative variation in the normalized voltage and current values $\Delta\bar{u}/\bar{u}$ and $\Delta\bar{i}/\bar{i}$, for a given value of $x = \ln(R_T/R_e)$ do not exceed the quantity γ, equal to

$$\gamma \simeq \frac{x}{2(1 - \Theta_\tau x)} \frac{\Delta T_e}{B} \tag{24}$$

(here $\Theta_\tau = \tau_e/B$). For an environment temperature change in the range $2\Delta T_e \le 100°C$, and for a thermistor excess temperature relative to the environment on the same order

$$\frac{x}{2(1-\Theta_\tau x)} \sim 1, \quad \frac{\Delta T_e}{B} \sim 10^{-2}$$

and thus the relative displacement γ of the normalized volt—ampere characteristic along the path $R_T = \bar{u}/\bar{i} =$ const (x = const) for an environment temperature change within the above range does not exceed a few percent.

From this we can derive the following technique for graphic-analytic design of circuits with thermoresistors:

1) We construct a normalized volt—ampere thermistor characteristic $\bar{u} = F(\bar{i}, \Theta_\tau)$, corresponding to the average environment temperature τ_e in the range of change;

2) using the equivalent generator theorem we calculate the voltage u_{eq} and the resistance r_{eq};

3) for the given specific values of environment temperature T_e and dissipation coefficient H_0, we calculate the thermoresistor resistance $R_e = f(T_e)$, the measurement scales for voltage M_u, and the normalized values of voltage $\bar{u}_{eq} = u_{eq}/M_u$ and resistance $\bar{r}_{eq} = r_{eq}/M_R$ ($M_R = M_u/M_i = R_e$ is the resistance measurement scale);

4) we draw the loadline $\bar{u}_{eq} - \bar{i}\,\bar{r}_{eq}$, determine the point A where it intersects the normalized volt—ampere characteristic, and the thermoresistor resistance at this point (first the normalized value $\bar{R}_A = \bar{u}_A/\bar{i}_A$ and then the ordinary dimensional value $R_A = \bar{R}_A R_e$). Knowing R_A, it is easy to determine the desired values of circuit current and voltage using the well-known relations for linear circuits.

Thus, in this design method, only the loadline is altered, and not the thermoresistor volt—ampere characteristic when the environment temperature T_e and the dissipation coefficient H_0 vary.

4. Thermoresistor Volt—Ampere Characteristics at an Extreme Point

The temperature T_m at an extreme point of the thermoresistor volt—ampere characteristic is given by the well-known expression

$$T_m = \frac{B}{2}\left(1 - \sqrt{1 - \frac{4T_e}{B}}\right)$$

or

$$\Theta_m = \frac{T_m}{B} = \frac{1}{2}(1 - \sqrt{1 - 4\Theta_e}). \tag{25}$$

The volt—ampere characteristics of thermistors have a voltage maximum only for $\Theta_e < 0.25$. Expanding the expression under the square root sign in Eq. (25) in a series in terms of $4\Theta_e$, we easily obtain the following relations for the normalized values of thermoresistor excess temperature and power dissipation at the extreme point:

$$\Delta\Theta_m = \Theta_m - \Theta_e = \Theta_e^2(1 + 2\Theta_e + 5\Theta_e^2 + 14\Theta_e^3 + \ldots), \tag{26}$$

$$P_m = H_m(T_m - T_e) = BH_m\Delta\Theta_m = BH_m\Theta_e^2(1 + 2\Theta_e + 5\Theta_e^2 + 14\Theta_e^3 + \ldots) \tag{27}$$

(H_m is the dissipation coefficient in the vicinity of the extreme point).

In practice, in most cases $\Theta_e \simeq 0.04$-0.15 ($B \simeq 3000$-6000°K, $T_e \simeq 250$-450°K), and the series in Eqs. (26)-(27) converge rapidly.

Having found the quantities $\Delta\Theta_m$ and P_m, and using the corresponding series expansions in terms of the parameter Θ_e, we can easily calculate R_m, U_m, and I_m:

$$R_m = R_e \exp\left(-\frac{T_m}{T_e}\right) = R_e \exp\left[-\left(1 + \frac{\Delta\Theta_m}{\Theta_e}\right)\right] = \frac{R_e}{e}\left(1 - \Theta_e - \frac{3}{2}\Theta_e^2 - \frac{19}{6}\Theta_e^3 - \ldots\right), \tag{28}$$

$$U_m = \sqrt{P_m R_m} = \Theta_e\sqrt{\frac{H_m B R_e}{e}}\left(1 + \frac{\Theta_e}{2} + \frac{5}{8}\Theta_e^2 + \frac{53}{48}\Theta_e^3 + \ldots\right). \tag{29}$$

(Eq. (29) for U_m was obtained in [10]),

$$I_m = \sqrt{\frac{P_m}{R_m}} = \Theta_e \sqrt{\frac{eH_mB}{R_e}} \left(1 - \frac{3}{2}\Theta_e - \frac{29}{8}\Theta_e^2 - \frac{487}{48}\Theta_e^3 - \ldots\right). \tag{30}$$

Equations (27)-(30) can be approximated by the following expressions:

$$P_m \simeq BH_m\Theta_e^2\left(1 - 2\frac{\Theta_e}{1-2.8\Theta_e}\right), \tag{31}$$

$$R_m \simeq \frac{R_e}{e}\left(1 - \frac{\Theta_e}{1-1.8\Theta_e}\right), \tag{32}$$

$$U_m \simeq \Theta_e \sqrt{\frac{BH_mR_e}{e}}\left(1 + \frac{1}{2}\cdot\frac{\Theta_e}{1-1.5\Theta_e}\right), \tag{33}$$

$$I_m \simeq \Theta_e \sqrt{\frac{eBH_m}{R_e}}\left(1 - \frac{3}{2}\cdot\frac{\Theta_e}{1-2.75\Theta_e}\right). \tag{34}$$

For a change in the parameter $\Theta_e = T_e/B$ in the range $0 \le \Theta_e \le 0.2$, the error in using Eq. (33) does not exceed 0.2%, and when using Eqs. (31), (32), and (34), the error does not exceed 1%, in comparison with the exact values of P_m, R_m, U_m, and I_m.

From the above relations (33) and (34) we can obtain a simple analytical expression for calculating the voltage $U_{r\,eq}$ corresponding to the onset of a relay effect in the thermoresistor arm, for $0 \le r_{eq} < |r_d|$:

$$U_{r\,eq} \simeq U_m + r_{eq}I_m \simeq \Theta_e \sqrt{\frac{BH_mR_e}{e}}\left[1 + \frac{1}{2}\cdot\frac{\Theta_e}{1-1.5\Theta_e} + e\frac{r_{eq}}{R_e}\left(1 + \frac{3}{2}\cdot\frac{\Theta_e}{1-2.75\Theta_e}\right)\right]. \tag{35}$$

It can be shown that the differential resistance at the knee point is

$$r_d \simeq -\frac{\sqrt{3}-1}{\sqrt{3}+1}(1-3\Theta_e)\exp(-\sqrt{3}). \tag{36}$$

Relation (35) can be used to design devices which use the relay effect in a circuit with a thermoresistor. The error in calculating $U_{r\,eq}$ using Eq. (35) with $0.03 \le \Theta_e < 0.25$ does not exceed ~ 2%.

NOTATION

T, T_e, temperatures of thermoresistor and the surrounding environment; R_T, R_e, thermoresistor resistances at temperatures T and T_e; u, voltage over the thermoresistor; i, current through the thermoresistor; T_m, thermoresistor temperature corresponding to an extreme point; Θ_m, Θ_e, normalized dimensionless values of temperatures T_m and T_e; x, auxiliary variable; $\bar{u}$, ι, $\bar{R}_T$, normalized values of voltage, current, and resistance; m_u (M_u), m_i (M_i), m_R, m_p, measurement scales for voltage, current, resistance, and power; u_{eq}, r_{eq}, values of voltage and resistance determining the current through the circuit branch containing the thermoresistor; U_m, I_m, R_m, P_m, voltage over the thermoresistor, current through it, thermoresistor resistance, and power generated in the thermoresistor, corresponding to the extreme point of the volt–ampere characteristic; r_d, differential thermoresistor resistance at the knee point of the volt–ampere characteristic; γ, relative displacement of the volt–ampere characteristic along the path $\bar{R}_T$ = const; $U_{r\,eq}$, the voltage u_{eq} corresponding to the onset of the relay effect in a circuit with a thermoresistor.

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MOLECULAR-KINETIC GENERALIZATION OF THE HEAT-TRANSFER EQUATION

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Starting from microscopic theory, the authors generalize the heat-conduction equation to the case where the gradients in molecular transport velocities vary appreciably over a mean free path.

To obtain a hydrodynamic description of a rarefied gas as a continuous medium one usually begins from the Boltzmann equations and uses the method of successive approximations to derive the equations of an ideal compressible fluid, the Navier–Stokes equations, the Barnett equations and the super-Barnett equations. However, in the work of Predvoditelev [1, 2] and Truesdell [3, 4] it was noted that equations of higher order than the Navier–Stokes equations still give a poor description of the behavior of a rarefied gas (at least no better than the Navier–Stokes equations). Several different approaches from this basis have been suggested. For example, in the work of Vallander [5, 6] a method was suggested for generalizing the Boltzmann equation, with subsequent transition to equations of hydrodynamic type. Several modifications of the Navier–Stokes equations have been proposed by Ladyzhenskaya [7].

Predvoditelev [1] generalized the Navier–Stokes hydrodynamic equations, using the Maxwell method [8] and starting from the molecular-kinetic basis of the hydrodynamic equations. The Maxwell approach to deriving the equations of motion of a viscous fluid from the kinetic theory of gases, in contrast with the method of deriving the hydrodynamic equations from the Boltzmann equations, as developed in the work of Enskog and Chapman [9], does not require knowledge of the distribution function and is based on the following assumption.

The transport velocities of the two colliding molecules are equal; this means that a continuum in motion has a filamentary structure, i.e., the minimum dimensions of the jets correspond to the mean distance between molecules. The first to give attention to the possibility of generalizing this hypothesis was Predvoditelev [1], who stressed that the physical situation corresponding to Maxwell's hypothesis will not hold for motion of a continuum at large enough speed near a wall or when vortices are generated. In addition, the breakdown of the Maxwell hypothesis that the molecular transport speeds are equal will be evident in motion of a rarefied gas, when the flow dimensions are comparable with the average distance between molecules. The Predvoditelev hypothesis was further developed in regard to generalization of the equations of hydrodynamics in the work of Bubnov [10, 11].

In the present paper the concept of work [1] is used to derive a generalized heat-conduction equation for a rarefied gas, when the gradients of the transport speeds vary appreciably over a molecular mean free path.

1. Derivation of the Basic Equation

To derive a generalized heat-conduction equation we begin from the energy equation, obtained from microscopic theory [12]:

$$3\rho\frac{Dq}{Dt}=-\frac{\partial}{\partial x}\rho(\overline{\xi^3}+\overline{\xi\eta^2}+\overline{\xi\zeta^2})-\frac{\partial}{\partial y}\rho(\overline{\eta^3}+\overline{\xi\eta^3}+\overline{\eta\zeta^2})$$

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